

## 反射テスト 積分 定積分 いろいろ 03

1. 次の計算をせよ. ( S 級 3 分 20 秒, A 級 4 分 30 秒, B 級 6 分, C 級 8 分 )

(1)  $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^3 x \, dx$

(2)  $\int_{\frac{1}{e}}^e (\log x)^2 \, dx$

2. 次の計算をせよ. ( S 級 3 分 20 秒, A 級 4 分 30 秒, B 級 6 分, C 級 8 分 )

(1)  $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sin^3 x \, dx$

(2)  $\int_{\frac{1}{e}}^e (1 + \log x)^2 \, dx$

# 反射テスト 積分 定積分 いろいろ 03 解答解説

1. 次の計算をせよ。(S級3分20秒, A級4分30秒, B級6分, C級8分)

$$(1) \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^3 x \, dx$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (1 - \sin^2 x) \cos x \, dx$$

$$t = \sin x \text{ とおくと } \frac{dt}{dx} = \cos x \Leftrightarrow \cos x \, dx = dt$$

$$\text{また, } x = \frac{\pi}{6} \text{ のとき } t = \frac{1}{2}, \quad x = \frac{\pi}{2} \text{ のとき } t = 1$$

$x$	$\frac{\pi}{6}$	$\rightarrow$	$\frac{\pi}{2}$
$t$	$\frac{1}{2}$	$\rightarrow$	1

$$\text{与式} = \int_{\frac{1}{2}}^1 (1 - t^2) \, dt$$

$$= \left[ t - \frac{1}{3} t^3 \right]_{\frac{1}{2}}^1$$

$$= 1 - \frac{1}{3} - \left\{ \frac{1}{2} - \frac{1}{3} \cdot \left( \frac{1}{2} \right)^3 \right\}$$

$$= \frac{2}{3} - \frac{11}{24}$$

$$= \frac{5}{24} \quad \dots \text{答え}$$

$$(2) \int_{\frac{1}{e}}^e (\log x)^2 \, dx$$

$$= \left[ x (\log x)^2 \right]_{\frac{1}{e}}^e - \int_{\frac{1}{e}}^e x \cdot (2 \log x) \cdot \frac{1}{x} \, dx$$

$$= \left[ x (\log x)^2 \right]_{\frac{1}{e}}^e - 2 \int_{\frac{1}{e}}^e \log x \, dx$$

$$= \left[ x (\log x)^2 - 2(x \log x - x) \right]_{\frac{1}{e}}^e$$

$$= \left\{ e (\log e)^2 - 2(e \log e - e) \right\}$$

$$- \left\{ \frac{1}{e} \left( \log \frac{1}{e} \right)^2 - 2 \left( \frac{1}{e} \log \frac{1}{e} - \frac{1}{e} \right) \right\}$$

$$= e - \frac{1}{e} + 2 \left( -\frac{1}{e} - \frac{1}{e} \right)$$

$$= e - \frac{5}{e} \quad \dots \text{答え}$$

☆別解

$$\cos 3x = 4 \cos^3 x - 3 \cos x \Leftrightarrow \cos^3 x = \frac{\cos 3x + 3 \cos x}{4}$$

$$\therefore \text{与式} = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos 3x + 3 \cos x}{4} \, dx$$

$$= \frac{1}{4} \left[ \frac{1}{3} \sin 3x + 3 \sin x \right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

2. 次の計算をせよ。(S級3分20秒, A級4分30秒, B級6分, C級8分)

$$(1) \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sin^3 x \, dx$$

$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (1 - \cos^2 x) \sin x \, dx$$

$$t = \cos x \text{ とおくと } \frac{dt}{dx} = -\sin x \Leftrightarrow \sin x \, dx = -dt$$

$$\text{また, } x = \frac{\pi}{3} \text{ のとき } t = \frac{1}{2}, \quad x = \frac{\pi}{2} \text{ のとき } t = 0$$

$x$	$\frac{\pi}{3}$	$\rightarrow$	$\frac{\pi}{2}$
$t$	$\frac{1}{2}$	$\rightarrow$	$0$

$$\text{与式} = \int_{\frac{1}{2}}^0 (1 - t^2) \cdot (-1) \, dt$$

$$= \int_0^{\frac{1}{2}} (1 - t^2) \, dt$$

$$= \left[ t - \frac{1}{3}t^3 \right]_0^{\frac{1}{2}}$$

$$= \left\{ \frac{1}{2} - \frac{1}{3} \cdot \left( \frac{1}{2} \right)^3 \right\} - \left( 0 - \frac{1}{3} \cdot 0^3 \right)$$

$$= \frac{11}{24} \quad \dots \text{答え}$$

$$(2) \int_{\frac{1}{e}}^e (1 + \log x)^2 \, dx$$

$$= \int_{\frac{1}{e}}^e \left\{ 1 + 2 \log x + (\log x)^2 \right\} \, dx$$

$$= \left[ x + 2(x \log x - x) + x (\log x)^2 \right]_{\frac{1}{e}}^e$$

$$- \int_{\frac{1}{e}}^e x \cdot 2(\log x) \cdot \frac{1}{x} \, dx$$

$$= \left[ x + 2(x \log x - x) + x (\log x)^2 \right]_{\frac{1}{e}}^e - 2 \int_{\frac{1}{e}}^e (\log x) \, dx$$

$$= \left[ x + 2(x \log x - x) + x (\log x)^2 - 2(x \log x - x) \right]_{\frac{1}{e}}^e$$

$$= \left[ x + x (\log x)^2 \right]_{\frac{1}{e}}^e$$

$$= \left\{ e + e (\log e)^2 \right\} - \left\{ \frac{1}{e} + \frac{1}{e} \left( \log \frac{1}{e} \right)^2 \right\}$$

$$= 2e - \frac{2}{e} \quad \dots \text{答え}$$

☆別解

$$\sin 3x = 3 \sin x - 4 \sin^3 x \Leftrightarrow \sin^3 x = \frac{3 \sin x - \sin 3x}{4}$$

$$\therefore \text{与式} = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{3 \sin x - \sin 3x}{4} \, dx$$

$$= \frac{1}{4} \left[ -3 \cos x + \frac{1}{3} \cos 3x \right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$$