

反射テスト 積分 定積分 いろいろ 02

1. 次の計算をせよ。(S級2分50秒, A級4分20秒, B級7分, C級10分)

$$(1) \int_1^2 \frac{1}{x(x+1)} dx$$

$$(2) \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} (\sin x + \sin^2 x) dx$$

$$(3) \int_{\log 2}^{\log 8} x e^x dx$$

$$(4) \int_{-3}^3 \sqrt{36 - x^2} dx$$

2. 次の計算をせよ。(S級4分, A級6分, B級9分, C級13分)

$$(1) \int_2^4 \frac{1}{x^2 - 1} dx$$

$$(2) \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} (\tan x + \tan^2 x) dx$$

$$(3) \int_{\log 3}^{\log 9} x e^{-x} dx$$

$$(4) \int_{-2\sqrt{3}}^{2\sqrt{3}} \sqrt{16 - x^2} dx$$

反射テスト 積分 定積分 いろいろ 02 解答解説

1. 次の計算をせよ。(S級2分50秒, A級4分20秒, B級7分, C級10分)

$$(1) \int_1^2 \frac{1}{x(x+1)} dx$$

$$= \int_1^2 \left(\frac{1}{x} - \frac{1}{x+1} \right) dx$$

$$= [\log|x| - \log|x+1|]_1^2$$

$$= (\log 2 - \log 3) - (\log 1 - \log 2)$$

$$= 2 \log 2 - \log 3 \quad \dots \text{答え}$$

$$= \log \frac{4}{3} \quad \dots \text{答え}$$

$$(2) \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} (\sin x + \sin^2 x) dx$$

$$= \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \sin^2 x dx$$

$$= 2 \int_0^{\frac{\pi}{6}} \sin^2 x dx \quad \leftarrow \star \text{偶関数の定積分}$$

$$= 2 \cdot \frac{1}{2} \int_0^{\frac{\pi}{6}} (1 - \cos 2x) dx$$

$$= \left[x - \frac{1}{2} \sin 2x \right]_0^{\frac{\pi}{6}}$$

$$= \left(\frac{\pi}{6} - \frac{\sqrt{3}}{4} \right) - (0 - 0)$$

$$= \frac{\pi}{6} - \frac{\sqrt{3}}{4} \quad \dots \text{答え}$$

$$(3) \int_{\log 2}^{\log 8} x e^x dx$$

$$= [x \cdot e^x]_{\log 2}^{\log 8} - \int_{\log 2}^{\log 8} (x)' \cdot e^x dx \quad \leftarrow \star$$

$$= \log 8 \cdot e^{\log 8} - \log 2 \cdot e^{\log 2} - \int_{\log 2}^{\log 8} e^x dx$$

$$= 8 \log 2^3 - 2 \log 2 - [e^x]_{\log 2}^{\log 8}$$

$$= 24 \log 2 - 2 \log 2 - (e^{\log 8} - e^{\log 2})$$

$$= 22 \log 2 - (8 - 2)$$

$$= 22 \log 2 - 6 \quad \dots \text{答え}$$

★部分積分

$$(4) \int_{-3}^3 \sqrt{36 - x^2} dx$$

★扇形の面積で考える

$y = \sqrt{36 - x^2}$ ($-3 \leq x \leq 3$) は半径6の円周.

∴ 与式 = (六分円 + 三角形 × 2) の面積

$$\text{与式} = \pi \cdot 6^2 \cdot \frac{1}{6} + 3 \times 3\sqrt{3} \times \frac{1}{2} \times 2$$

$$= 6\pi + 9\sqrt{3} \quad \dots \text{答え}$$

☆別解 ($x = 6 \sin \theta$ で置換)

$$\text{与式} = 2 \int_0^3 \sqrt{6^2 - x^2} dx$$

$$= 2 \int_0^{\frac{\pi}{6}} 6 |\cos \theta| \cdot 6 \cos \theta d\theta$$

$$= 2 \cdot 36 \cdot \frac{1}{2} \int_0^{\frac{\pi}{6}} (1 + \cos 2\theta) d\theta$$

$$= 36 \left[\theta + \frac{1}{2} \sin 2\theta \right]_0^{\frac{\pi}{6}} = 6\pi + 9\sqrt{3}$$

2. 次の計算をせよ。(S級4分, A級6分, B級9分, C級13分)

$$(1) \int_2^4 \frac{1}{x^2-1} dx$$

$$\begin{aligned} &= \frac{1}{2} \int_2^4 \left(\frac{1}{x-1} - \frac{1}{x+1} \right) dx \\ &= \frac{1}{2} [\log|x-1| - \log|x+1|]_2^4 \\ &= \frac{1}{2} \{(\log 3 - \log 5) - (\log 1 - \log 3)\} \\ &= \log 3 - \frac{1}{2} \log 5 \quad \cdots \text{答え} \end{aligned}$$

$$(2) \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} (\tan x + \tan^2 x) dx$$

$$\begin{aligned} &= \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \tan^2 x dx \\ &= 2 \int_0^{\frac{\pi}{3}} \tan^2 x dx \quad \leftarrow \star \text{偶関数の定積分} \\ &= 2 \int_0^{\frac{\pi}{3}} \left(\frac{1}{\cos^2 x} - 1 \right) dx \quad \leftarrow \star \\ &= 2 [\tan x - x]_0^{\frac{\pi}{3}} \\ &= 2 \left\{ \left(\sqrt{3} - \frac{\pi}{3} \right) - (0 - 0) \right\} \\ &= 2\sqrt{3} - \frac{2}{3}\pi \quad \cdots \text{答え} \end{aligned}$$

$$\begin{aligned} \star 1 + \tan^2 x &= \frac{1}{\cos^2 x} \\ \Leftrightarrow \tan^2 x &= \frac{1}{\cos^2 x} - 1 \end{aligned}$$

$$(3) \int_{\log 3}^{\log 9} x e^{-x} dx$$

$$\begin{aligned} &= [x \cdot (-e^{-x})]_{\log 3}^{\log 9} - \int_{\log 3}^{\log 9} (x)' \cdot (-e^{-x}) dx \quad \leftarrow \star \\ &= -[x e^{-x}]_{\log 3}^{\log 9} + \int_{\log 3}^{\log 9} e^{-x} dx \\ &= -\log 9 \cdot e^{-\log 9} + \log 3 \cdot e^{-\log 3} + \int_{\log 3}^{\log 9} e^{-x} dx \\ &= -\frac{1}{9} \log 3^2 + \frac{1}{3} \log 3 + [-e^{-x}]_{\log 3}^{\log 9} \\ &= \frac{1}{9} \log 3 - (e^{-\log 9} - e^{-\log 3}) \\ &= \frac{1}{9} \log 3 - \frac{1}{9} + \frac{1}{3} = \frac{1}{9} \log 3 + \frac{2}{9} \quad \cdots \text{答え} \end{aligned}$$

★部分積分

$$(4) \int_{-2\sqrt{3}}^{2\sqrt{3}} \sqrt{16-x^2} dx$$

★扇形の面積で考える
 $y = \sqrt{16-x^2}$ ($-2\sqrt{3} \leq x \leq 2\sqrt{3}$) は半径4の円周。

∴ 与式 = (三分円と三角形 × 2) の面積

$$\begin{aligned} \text{与式} &= \pi \cdot 4^2 \cdot \frac{1}{3} + 2 \times 2\sqrt{3} \times \frac{1}{2} \times 2 \\ &= \frac{16}{3}\pi + 4\sqrt{3} \quad \cdots \text{答え} \end{aligned}$$

☆別解 ($x = 4 \sin \theta$ で置換)

$$\begin{aligned} \text{与式} &= 2 \int_0^{2\sqrt{3}} \sqrt{4^2 - x^2} dx \\ &= 2 \int_0^{\frac{\pi}{3}} 4|\cos \theta| \cdot 4 \cos \theta d\theta \\ &= 2 \cdot 16 \cdot \frac{1}{2} \int_0^{\frac{\pi}{3}} (1 + \cos 2\theta) d\theta \\ &= 16 \left[\theta + \frac{1}{2} \sin 2\theta \right]_0^{\frac{\pi}{3}} = \frac{16}{3}\pi + 4\sqrt{3} \end{aligned}$$