

# 反射テスト 積分 定積分 いろいろ 01

1. 次の計算をせよ。(S級1分40秒, A級2分30秒, B級3分, C級5分)

$$(1) \int_1^{e^2} \frac{1}{x} dx$$

$$(2) \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} (5x + 2 \sin x) dx$$

$$(3) \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos^2 x dx$$

$$(4) \int_1^e x \log x dx$$

2. 次の計算をせよ。(S級3分40秒, A級5分, B級7分, C級10分)

$$(1) \int_4^{32} \frac{1}{2x} dx$$

$$(2) \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (x^3 + x^2 + x + \cos x) dx$$

$$(3) \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \sin^2 x dx$$

$$(4) \int_1^8 x \log \frac{x}{4} dx$$

# 反射テスト 積分 定積分 いろいろ 01 解答解説

1. 次の計算をせよ。(S級1分40秒, A級2分30秒, B級3分, C級5分)

$$(1) \int_1^{e^2} \frac{1}{x} dx$$

$$= [\log |x|]_1^{e^2}$$

$$= \log e^2 - \log 1$$

$$= 2 - 0 = 2 \quad \dots \text{答え}$$

$$(2) \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} (5x + 2 \sin x) dx$$

$$= 0 \quad \dots \text{答え}$$

★奇関数の定積分

$$(3) \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos^2 x dx$$

$$= 2 \int_0^{\frac{\pi}{4}} \frac{1 + \cos 2x}{2} dx \quad \leftarrow \text{★偶関数の定積分}$$

$$= 2 \cdot \frac{1}{2} \int_0^{\frac{\pi}{4}} (1 + \cos 2x) dx$$

$$= \left[ x + \frac{1}{2} \sin 2x \right]_0^{\frac{\pi}{4}}$$

$$= \left( \frac{\pi}{4} + \frac{1}{2} \cdot 1 \right) - (0 + 0)$$

$$= \frac{\pi}{4} + \frac{1}{2} \quad \dots \text{答え}$$

$$(4) \int_1^e x \log x dx$$

$$= \int_1^e \left( \frac{1}{2} x^2 \right)' \cdot \log x dx$$

$$= \left[ \frac{1}{2} x^2 \cdot \log x \right]_1^e - \int_1^e \frac{1}{2} x^2 \cdot (\log x)' dx \quad \leftarrow \text{★}$$

$$= \left[ \frac{1}{2} x^2 \log x \right]_1^e - \int_1^e \frac{1}{2} x^2 \cdot \frac{1}{x} dx \quad \leftarrow \text{☆}$$

$$= \left[ \frac{1}{2} x^2 \log x \right]_1^e - \frac{1}{2} \int_1^e x dx$$

$$= \left[ \frac{1}{2} x^2 \log x - \frac{1}{4} x^2 \right]_1^e$$

$$= \left( \frac{1}{2} \cdot e^2 \log e - \frac{1}{4} \cdot e^2 \right) - \left( \frac{1}{2} \cdot 1^2 \log 1 - \frac{1}{4} \cdot 1^2 \right)$$

$$= \frac{1}{4} e^2 + \frac{1}{4} \quad \dots \text{答え}$$

★部分積分

☆  $\because x > 0 \Rightarrow |x| = x$

2. 次の計算をせよ。(S級3分40秒, A級5分, B級7分, C級10分)

$$(1) \int_4^{32} \frac{1}{2x} dx$$

$$\begin{aligned} &= \frac{1}{2} [\log |x|]_4^{32} \\ &= \frac{1}{2} (\log 32 - \log 4) \\ &= \frac{1}{2} (5 \log 2 - 2 \log 2) \\ &= \frac{3}{2} \log 2 \quad \dots \text{答え} \end{aligned}$$

$$(2) \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (x^3 + x^2 + x + \cos x) dx$$

$$\begin{aligned} &= 2 \int_0^{\frac{\pi}{4}} (x^2 + \cos x) dx \quad \leftarrow \star \text{奇関数} \cdot \text{偶関数の定積分} \\ &= 2 \left[ \frac{1}{3} x^3 + \sin x \right]_0^{\frac{\pi}{4}} \\ &= \frac{2}{3} \left( \frac{\pi}{4} \right)^3 + 2 \cdot \frac{1}{\sqrt{2}} \\ &= \frac{\pi^3}{96} + \sqrt{2} \quad \dots \text{答え} \end{aligned}$$

$$(3) \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \sin^2 x dx$$

$$\begin{aligned} &= 2 \int_0^{\frac{\pi}{3}} \frac{1 - \cos 2x}{2} dx \quad \star \text{偶関数の定積分} \\ &= 2 \cdot \frac{1}{2} \int_0^{\frac{\pi}{3}} (1 - \cos 2x) dx \\ &= \left[ x - \frac{1}{2} \sin 2x \right]_0^{\frac{\pi}{3}} \\ &= \left( \frac{\pi}{3} - \frac{1}{2} \cdot \frac{\sqrt{3}}{2} \right) - (0 - 0) \\ &= \frac{\pi}{3} - \frac{\sqrt{3}}{4} \quad \dots \text{答え} \end{aligned}$$

$$(4) \int_1^8 x \log \frac{x}{4} dx$$

$$\begin{aligned} &= \int_1^8 \left( \frac{1}{2} x^2 \right)' \cdot \log \frac{x}{4} dx \\ &= \left[ \frac{1}{2} x^2 \cdot \log \frac{x}{4} \right]_1^8 - \int_1^8 \frac{1}{2} x^2 \cdot \left( \log \frac{x}{4} \right)' dx \quad \leftarrow \star \\ &= \left[ \frac{1}{2} x^2 \log \frac{x}{4} \right]_1^8 - \int_1^8 \frac{1}{2} x^2 \cdot \frac{1}{4} \cdot \frac{4}{x} dx \quad \leftarrow \star \\ &= \left[ \frac{1}{2} x^2 \log \frac{x}{4} \right]_1^8 - \frac{1}{2} \int_1^8 x dx \\ &= \left[ \frac{1}{2} x^2 \log \frac{x}{4} - \frac{1}{4} x^2 \right]_1^8 \\ &= \left( \frac{1}{2} \cdot 8^2 \log 2 - \frac{1}{4} \cdot 8^2 \right) - \left( \frac{1}{2} \cdot 1^2 \log \frac{1}{4} - \frac{1}{4} \cdot 1^2 \right) \\ &= 32 \log 2 - 16 - \left( -\log 2 - \frac{1}{4} \right) \\ &= 33 \log 2 - \frac{63}{4} \quad \dots \text{答え} \end{aligned}$$

★部分積分

☆  $\because x > 0 \Rightarrow |x| = x$