

## 反射テスト 積分 定積分 部分積分法 01

1. 次の計算をせよ。(S級1分30秒, A級2分40秒, B級4分, C級5分30秒)

(1)  $\int_0^1 x e^x dx$

(2)  $\int_1^e x \log x dx$

2. 次の計算をせよ. ( S 級 2 分, A 級 3 分 20 秒, B 級 5 分, C 級 7 分 )

(1)  $\int_0^1 x e^{2x-1} dx$

(2)  $\int_1^e x^2 \log x dx$

# 反射テスト 積分 定積分 部分積分法 01 解答解説

1. 次の計算をせよ。(S級1分30秒, A級2分40秒, B級4分, C級5分30秒)

★ 部分積分法  $\int_a^b f(x)g'(x) dx = [f(x)g(x)]_a^b - \int_a^b f'(x)g(x) dx$

$$\begin{aligned} (1) \quad & \int_0^1 x e^x dx \\ &= \int_0^1 x \cdot (e^x)' dx \\ &= [x e^x]_0^1 - \int_0^1 (x)' \cdot e^x dx \\ &= [x e^x]_0^1 - \int_0^1 e^x dx \\ &= [x e^x]_0^1 - [e^x]_0^1 \\ &= [(x-1)e^x]_0^1 \\ &= 0 - (-1)e^0 = \mathbf{1} \end{aligned}$$

$$\begin{aligned} (2) \quad & \int_1^e x \log x dx \\ &= \int_1^e \left(\frac{1}{2}x^2\right)' \cdot \log x dx \\ &= \left[\frac{1}{2}x^2 \cdot \log x\right]_1^e - \int_1^e \frac{1}{2}x^2 \cdot (\log x)' dx \\ &= \left[\frac{1}{2}x^2 \log x\right]_1^e - \int_1^e \frac{1}{2}x^2 \cdot \frac{1}{x} dx \\ &= \left[\frac{1}{2}x^2 \log x\right]_1^e - \frac{1}{2} \int_1^e x dx \\ &= \left[\frac{1}{2}x^2 \log x - \frac{1}{4}x^2\right]_1^e \\ &= \left(\frac{1}{2}e^2 \log e - \frac{1}{4}e^2\right) - \left(\frac{1}{2}1^2 \log 1 - \frac{1}{4}1^2\right) \\ &= \frac{1}{4}e^2 + \frac{1}{4} \end{aligned}$$

2. 次の計算をせよ。(S級2分, A級3分20秒, B級5分, C級7分)

$$\begin{aligned}(1) \quad & \int_0^1 x e^{2x-1} dx \\ &= \int_0^1 x \cdot \left(\frac{1}{2} e^{2x-1}\right)' dx \\ &= \left[x \cdot \frac{1}{2} e^{2x-1}\right]_0^1 - \int_0^1 (x)' \cdot \frac{1}{2} e^{2x-1} dx \\ &= \frac{1}{2} [x e^{2x-1}]_0^1 - \frac{1}{2} \int_0^1 e^{2x-1} dx \\ &= \frac{1}{2} [x e^{2x-1}]_0^1 - \frac{1}{4} [e^{2x-1}]_0^1 \\ &= \frac{1}{4} [(2x-1)e^{2x-1}]_0^1 \\ &= \frac{1}{4} \{e - (-1 \cdot e^{-1})\} \\ &= \frac{1}{4} \left(e + \frac{1}{e}\right)\end{aligned}$$

$$\begin{aligned}(2) \quad & \int_1^e x^2 \log x dx \\ &= \int_1^e \left(\frac{1}{3} x^3\right)' \cdot \log x dx \\ &= \left[\frac{1}{3} x^3 \cdot \log x\right]_1^e - \int_1^e \frac{1}{3} x^3 \cdot (\log x)' dx \\ &= \left[\frac{1}{3} x^3 \log x\right]_1^e - \int_1^e \frac{1}{3} x^3 \cdot \frac{1}{x} dx \\ &= \left[\frac{1}{3} x^3 \log x\right]_1^e - \frac{1}{3} \int_1^e x^2 dx \\ &= \left[\frac{1}{3} x^3 \log x - \frac{1}{9} x^3\right]_1^e \\ &= \left(\frac{1}{3} e^3 \log e - \frac{1}{9} e^3\right) - \left(\frac{1}{3} 1^3 \log 1 - \frac{1}{9} 1^3\right) \\ &= \frac{2}{9} e^3 + \frac{1}{9}\end{aligned}$$