

反射テスト 微分 いろいろ 01

1. $\frac{dy}{dx}$ を求めよ. (S 級 1 分, A 級 2 分, B 級 3 分, C 級 5 分)

(1) $y = \frac{3x}{x^2 + 1}$

(2) $y = \sin x^2$

(3) $y = \log |3x - 4|$

(4) $y = 2^{x^3+1}$

2. $\frac{dy}{dx}$ を求めよ。(S級1分, A級2分20秒, B級3分30秒, C級6分)

(1) $y = \frac{2x^2}{2x^2 + 1}$

(2) $y = \tan x^2$

(3) $y = \log |\cos 3x|$

(4) $y = 3^{\sin x}$

反射テスト 微分 いろいろ 01 解答解説

1. $\frac{dy}{dx}$ を求めよ。(S級1分, A級2分, B級3分, C級5分)

(1) $y = \frac{3x}{x^2 + 1}$

$$\begin{aligned} y' &= \frac{(3x)' \cdot (x^2 + 1) - (3x) \cdot (x^2 + 1)'}{(x^2 + 1)^2} \\ &= \frac{3(x^2 + 1) - 3x \cdot 2x}{(x^2 + 1)^2} \\ &= \frac{-3x^2 + 3}{(x^2 + 1)^2} \quad \dots\text{答え} \\ &= \frac{-3(x+1)(x-1)}{(x^2 + 1)^2} \quad \dots\text{答え} \end{aligned}$$

(2) $y = \sin x^2$

$$\begin{aligned} y' &= (\sin x^2)' \\ &= (\cos x^2) \cdot 2x \\ &= 2x \cos x^2 \quad \dots\text{答え} \end{aligned}$$

(3) $y = \log |3x - 4|$

$$\begin{aligned} y' &= (\log |3x - 4|)' \\ &= \frac{1}{3x - 4} \cdot (3x - 4)' \\ &= \frac{3}{3x - 4} \quad \dots\text{答え} \end{aligned}$$

★ 対数微分法

$$\{\log |f(x)|\}' = \frac{f'(x)}{f(x)}$$

絶対値で心配があれば,

$y = \log(4 - 3x)$ を微分してみるといい.

(4) $y = 2^{(x^3+1)}$

y は, x を 2 乗して 1 を足して,
3 についてのそのべき乗をとったもの.

$$\Rightarrow \begin{cases} u = x^3 + 1 \\ y = 2^u \end{cases} \quad \text{とおけばよい.}$$

$$\begin{aligned} y &= 2^u \quad u = x^3 + 1 \\ \Rightarrow \frac{dy}{du} &= 2^u \log 2 \quad \frac{du}{dx} = 3x^2 \end{aligned}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du} \cdot \frac{du}{dx} \\ &= 2^u \log 2 \cdot 3x^2 \\ &= 2^{(x^3+1)} \cdot \log 2 \cdot 3x^2 \\ &= 3x^2 \cdot 2^{(x^3+1)} \cdot \log 2 \quad \dots\text{答え} \end{aligned}$$

2. $\frac{dy}{dx}$ を求めよ。(S級1分, A級2分20秒, B級3分30秒, C級6分)

$$(1) \quad y = \frac{2x^2}{2x^2 + 1}$$

$$\begin{aligned} y' &= \frac{(2x^2)' \cdot (2x^2 + 1) - (2x^2) \cdot (2x^2 + 1)'}{(2x^2 + 1)^2} \\ &= \frac{4x(2x^2 + 1) - 2x^2 \cdot 4x}{(2x^2 + 1)^2} \\ &= \frac{4x}{(2x^2 + 1)^2} \quad \dots \text{答え} \end{aligned}$$

$$(2) \quad y = \tan x^2$$

$$\begin{aligned} y' &= (\tan x^2)' \\ &= \frac{1}{\cos^2 x^2} \cdot 2x \\ &= \frac{2x}{\cos^2 x^2} \quad \dots \text{答え} \end{aligned}$$

$$(3) \quad y = \log |\cos 3x|$$

$$\begin{aligned} y' &= (\log |\cos 3x|)' \\ &= \frac{(\cos 3x)'}{\cos 3x} \\ &= \frac{-3 \sin 3x}{\cos 3x} \\ &= -3 \tan 3x \quad \dots \text{答え} \end{aligned}$$

$$(4) \quad y = 3^{\sin x}$$

$$\Rightarrow \begin{cases} u = \sin x \\ y = 3^u \end{cases} \quad \text{とおく.}$$

$$\begin{aligned} y &= 3^u \quad u = \sin x \\ \Rightarrow \frac{dy}{du} &= 3^u \log 3 \quad \frac{du}{dx} = \cos x \end{aligned}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du} \cdot \frac{du}{dx} \\ &= 3^u \log 3 \cdot \cos x \\ &= 3^{\sin x} \cdot \log 3 \cdot \cos x \\ &= 3^{\sin x} \cdot \cos x \cdot \log 3 \quad \dots \text{答え} \end{aligned}$$