

反射テスト 微分 商の導関数 01

1. $\frac{dy}{dx}$ を求めよ. (S 級 2 分, A 級 3 分 20 秒, B 級 5 分, C 級 7 分)

(1) $y = \frac{x+2}{x-1}$

(2) $y = \frac{2x+1}{4-x}$

(3) $y = \frac{3x}{x^2+1}$

(4) $y = \frac{7-2x}{1-x^2}$

(5) $y = \frac{\log x}{x}$

(6) $y = \frac{\cos x}{\sin x}$

2. $\frac{dy}{dx}$ を求めよ。(S級2分30秒, A級4分, B級6分, C級9分)

(1) $y = \frac{x}{x+7}$

(2) $y = \frac{5x+2}{3-2x}$

(3) $y = \frac{x-2}{1-x^2}$

(4) $y = \frac{1-x}{x^3+1}$

(5) $y = \frac{e^x}{x}$

(6) $y = \frac{\log x}{\tan x}$

反射テスト 微分 商の導関数 01 解答解説

1. $\frac{dy}{dx}$ を求めよ。(S級2分, A級3分20秒, B級5分, C級7分)

★ 商の導関数
$$\begin{cases} \left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \\ \left(\frac{1}{v}\right)' = -\frac{v'}{v^2} \end{cases}$$

(1) $y = \frac{x+2}{x-1}$

$$\begin{aligned} y' &= \frac{(x+2)' \cdot (x-1) - (x+2) \cdot (x-1)'}{(x-1)^2} \\ &= \frac{(x-1) - (x+2)}{(x-1)^2} \\ &= -\frac{3}{(x-1)^2} \quad \dots\text{答え} \end{aligned}$$

(2) $y = \frac{2x+1}{4-x}$

$$\begin{aligned} y' &= \frac{(2x+1)' \cdot (4-x) - (2x+1) \cdot (4-x)'}{(4-x)^2} \\ &= \frac{2(4-x) - (2x+1) \cdot (-1)}{(4-x)^2} \\ &= \frac{8-2x+2x+1}{(4-x)^2} \\ &= \frac{9}{(x-4)^2} \quad \dots\text{答え} \end{aligned}$$

(3) $y = \frac{3x}{x^2+1}$

$$\begin{aligned} y' &= \frac{(3x)' \cdot (x^2+1) - (3x) \cdot (x^2+1)'}{(x^2+1)^2} \\ &= \frac{3(x^2+1) - 3x \cdot 2x}{(x^2+1)^2} \\ &= \frac{-3x^2+3}{(x^2+1)^2} \quad \dots\text{答え} \\ &= \frac{-3(x+1)(x-1)}{(x^2+1)^2} \quad \dots\text{答え} \end{aligned}$$

(4) $y = \frac{7-2x}{1-x^2}$

$$\begin{aligned} y' &= \frac{(7-2x)' \cdot (1-x^2) - (7-2x) \cdot (1-x^2)'}{(1-x^2)^2} \\ &= \frac{-2(1-x^2) - (7-2x) \cdot (-2x)}{(x^2-1)^2} \\ &= \frac{-2+2x^2+14x-4x^2}{(x^2-1)^2} \\ &= \frac{-2x^2+14x-2}{(x^2-1)^2} \quad \dots\text{答え} \\ &= -\frac{2(x^2-7x+1)}{(x^2-1)^2} \quad \dots\text{答え} \end{aligned}$$

(5) $y = \frac{\log x}{x}$

$$\begin{aligned} y' &= \frac{(\log x)' \cdot x - \log x \cdot (x)'}{x^2} \\ &= \frac{\frac{1}{x} \cdot x - \log x \cdot 1}{x^2} \\ &= \frac{1 - \log x}{x^2} \quad \dots\text{答え} \end{aligned}$$

(6) $y = \frac{\cos x}{\sin x}$

$$\begin{aligned} y' &= \frac{(\cos x)' \cdot \sin x - \cos x \cdot (\sin x)'}{\sin^2 x} \\ &= \frac{-\sin x \cdot \sin x - \cos x \cdot \cos x}{\sin^2 x} \\ &= -\frac{1}{\sin^2 x} \quad \dots\text{答え} \end{aligned}$$

次を公式として覚えておくこと.

★ $(\tan x)' = \frac{1}{\cos^2 x}$

★ $\left(\frac{1}{\tan x}\right)' = -\frac{1}{\sin^2 x}$

2. $\frac{dy}{dx}$ を求めよ。(S級2分30秒, A級4分, B級6分, C級9分)

$$(1) \quad y = \frac{x}{x+7}$$

$$\begin{aligned} y' &= \frac{(x)' \cdot (x+7) - x \cdot (x+7)'}{(x+7)^2} \\ &= \frac{x+7-x}{(x+7)^2} \\ &= \frac{7}{(x+7)^2} \quad \dots\text{答え} \end{aligned}$$

$$(2) \quad y = \frac{5x+2}{3-2x}$$

$$\begin{aligned} y' &= \frac{(5x+2)' \cdot (3-2x) - (5x+2) \cdot (3-2x)'}{(3-2x)^2} \\ &= \frac{5(3-2x) - (5x+2) \cdot (-2)}{(2x-3)^2} \\ &= \frac{15-10x+10x+4}{(2x-3)^2} \\ &= \frac{19}{(2x-3)^2} \quad \dots\text{答え} \end{aligned}$$

$$(3) \quad y = \frac{x-2}{1-x^2}$$

$$\begin{aligned} y' &= \frac{(x-2)' \cdot (1-x^2) - (x-2) \cdot (1-x^2)'}{(1-x^2)^2} \\ &= \frac{1-x^2 - (x-2) \cdot (-2x)}{(x^2-1)^2} \\ &= \frac{1-x^2+2x^2-4x}{(x^2-1)^2} \\ &= \frac{x^2-4x+1}{(x^2-1)^2} \quad \dots\text{答え} \end{aligned}$$

$$(4) \quad y = \frac{1-x}{x^3+1}$$

$$\begin{aligned} y' &= \frac{(1-x)' \cdot (x^3+1) - (1-x) \cdot (x^3+1)'}{(x^3+1)^2} \\ &= \frac{-1(x^3+1) - (1-x) \cdot 3x^2}{(x^3+1)^2} \\ &= \frac{-x^3-1-3x^2+3x^3}{(x^3+1)^2} \\ &= \frac{2x^3-3x^2-1}{(x^3+1)^2} \quad \dots\text{答え} \end{aligned}$$

$$(5) \quad y = \frac{e^x}{x}$$

$$\begin{aligned} y' &= \frac{(e^x)' \cdot x - e^x \cdot (x)'}{x^2} \\ &= \frac{xe^x - e^x}{x^2} \quad \dots\text{答え} \\ &= \frac{(x-1)e^x}{x^2} \quad \dots\text{答え} \end{aligned}$$

$$(6) \quad y = \frac{\log x}{\tan x}$$

$$\begin{aligned} y' &= \frac{(\log x)' \cdot \tan x - \log x \cdot (\tan x)'}{\tan^2 x} \\ &= \frac{\frac{1}{x} \cdot \tan x - \log x \cdot \frac{1}{\cos^2 x}}{\tan^2 x} \\ &= \frac{\tan x \cos^2 x - x \log x}{x \cos^2 x \tan^2 x} \\ &= \frac{\sin x \cos x - x \log x}{x \sin^2 x} \quad \dots\text{答え} \end{aligned}$$