

反射テスト 微分 積の導関数 01

1. $\frac{dy}{dx}$ を求めよ. (S 級 2 分, A 級 3 分 20 秒, B 級 5 分, C 級 7 分)

(1) $y = (3x + 5)(2x - 1)$

(2) $y = (x^2 + 1)(2x^3 - 1)$

(3) $y = x \log x$

(4) $y = \sin x \cos x$

(5) $y = (x - 1)(x + 2)(x + 3)$

(6) $y = xe^x \log x$

2. $\frac{dy}{dx}$ を求めよ。(S級2分, A級3分20秒, B級5分, C級7分)

(1) $y = (2x - 5)(3x + 2)$

(2) $y = (x^4 + 3)(x^3 - 1)$

(3) $y = \sin x \log x$

(4) $y = e^x \tan x$

(5) $y = (1 - x)(2 - x)(3 - x)$

(6) $y = x^2 \cos x \log x$

反射テスト 微分 積の導関数 01 解答解説

1. $\frac{dy}{dx}$ を求めよ。(S級2分, A級3分20秒, B級5分, C級7分)

★積の導関数
$$\begin{cases} (uv)' = u'v + uv' \\ (uvw)' = (uv)'v + uv(w)' = u'vw + uv'w + uvw' \end{cases}$$

(1) $y = (3x + 5)(2x - 1)$

$$\begin{aligned} y' &= (3x + 5)' \cdot (2x - 1) + (3x + 5) \cdot (2x - 1)' \\ &= 3(2x - 1) + (3x + 5) \cdot 2 \\ &= 6x - 3 + 6x + 10 \\ &= 12x + 7 \quad \cdots\text{答え} \end{aligned}$$

(2) $y = (x^2 + 1)(2x^3 - 1)$

$$\begin{aligned} y' &= (x^2 + 1)' \cdot (2x^3 - 1) + (x^2 + 1) \cdot (2x^3 - 1)' \\ &= 2x \cdot (2x^3 - 1) + (x^2 + 1) \cdot 6x^2 \\ &= 4x^4 - 2x + 6x^4 + 6x^2 \\ &= 10x^4 + 6x^2 - 2x \quad \cdots\text{答え} \end{aligned}$$

☆実戦では展開して解くほうが早いかも。

(3) $y = x \log x$

$$\begin{aligned} y' &= (x)' \log x + x \cdot (\log x)' \\ &= \log x + x \cdot \frac{1}{x} \\ &= \log x + 1 \quad \cdots\text{答え} \end{aligned}$$

(4) $y = \sin x \cos x$

$$\begin{aligned} y' &= (\sin x)' \cos x + \sin x \cdot (\cos x)' \\ &= \cos x \cdot \cos x + \sin x \cdot (-\sin x) \\ &= \cos^2 x - \sin^2 x \quad \cdots\text{答え} \\ &= \cos 2x \quad \cdots\text{答え} \end{aligned}$$

☆ $\cos 2x$ まで変形するほうがよい。

☆ $\sin 2x = 2 \sin x \cos x$ であるから、

$y = \frac{1}{2} \sin 2x$ として微分するのもよい。

(5) $y = (x - 1)(x + 2)(x + 3)$

$$\begin{aligned} y' &= (x - 1)'(x + 2)(x + 3) + (x - 1)(x + 2)'(x + 3) \\ &\quad + (x - 1)(x + 2)(x + 3)' \\ &= (x + 2)(x + 3) + (x - 1)(x + 3) + (x - 1)(x + 2) \\ &= x^2 + 5x + 6 + x^2 + 2x - 3 + x^2 + x - 2 \\ &= 3x^2 + 8x + 1 \quad \cdots\text{答え} \end{aligned}$$

(6) $y = xe^x \log x$

$$\begin{aligned} y' &= (x)'e^x \log x + x(e^x)' \log x + xe^x(\log x)' \\ &= e^x \log x + xe^x \log x + xe^x \cdot \frac{1}{x} \\ &= e^x (\log x + x \log x + 1) \quad \cdots\text{答え} \end{aligned}$$

☆対数微分法が早い。

2. $\frac{dy}{dx}$ を求めよ。(S級2分, A級3分20秒, B級5分, C級7分)

(1) $y = (2x - 5)(3x + 2)$

$$\begin{aligned}y' &= (2x - 5)' \cdot (3x + 2) + (2x - 5) \cdot (3x + 2)' \\&= 2(3x + 2) + (2x - 5) \cdot 3 \\&= 6x + 4 + 6x - 15 \\&= 12x - 11 \quad \cdots\text{答え}\end{aligned}$$

(2) $y = (x^4 + 3)(x^3 - 1)$

$$\begin{aligned}y' &= (x^4 + 3)' \cdot (x^3 - 1) + (x^4 + 3) \cdot (x^3 - 1)' \\&= 4x^3 \cdot (x^3 - 1) + (x^4 + 3) \cdot 3x^2 \\&= 4x^6 - 4x^3 + 3x^6 + 9x^2 \\&= 7x^6 - 4x^3 + 9x^2 \quad \cdots\text{答え}\end{aligned}$$

(3) $y = \sin x \log x$

$$\begin{aligned}y' &= (\sin x)' \log x + \sin x \cdot (\log x)' \\&= \cos x \log x + \sin x \cdot \frac{1}{x} \\&= \cos x \log x + \frac{\sin x}{x} \quad \cdots\text{答え}\end{aligned}$$

(4) $y = e^x \tan x$

$$\begin{aligned}y' &= (e^x)' \tan x + e^x \cdot (\tan x)' \\&= e^x \tan x + e^x \cdot \left(\frac{1}{\cos^2 x} \right) \\&= e^x \tan x + \frac{e^x}{\cos^2 x} \quad \cdots\text{答え}\end{aligned}$$

(5) $y = (1 - x)(2 - x)(3 - x)$

$$\begin{aligned}y' &= (1 - x)'(2 - x)(3 - x) + (1 - x)(2 - x)'(3 - x) \\&\quad + (1 - x)(2 - x)(3 - x)' \\&= -(2 - x)(3 - x) - (1 - x)(3 - x) - (1 - x)(2 - x) \\&= -(x^2 - 5x + 6) - (x^2 - 4x + 3) - (x^2 - 3x + 2) \\&= -3x^2 + 12x - 11 \quad \cdots\text{答え}\end{aligned}$$

(6) $y = x^2 \cos x \log x$

$$\begin{aligned}y' &= (x^2)' \cos x \log x + x^2(\cos x)' \log x + x^2 \cos x(\log x)' \\&= 2x \cos x \log x + x^2 \cdot (-\sin x) \log x + x^2 \cos x \cdot \frac{1}{x} \\&= 2x \cos x \log x - x^2 \sin x \log x + x \cos x \quad \cdots\text{答え}\end{aligned}$$

☆対数微分法が早い。