反射テスト　ベクトル　内積から角度を求める　01

1.　$\angle\mathrm{AOB}$ は何度か求めよ.（ S 級 1 分, A 級 2 分, B 級 3 分, C 級 5 分 ）

(1)　$\begin{cases} \mathrm{OA} = 3 \\ \mathrm{OB} = 6 \\ \overrightarrow{\mathrm{OA}} \cdot \overrightarrow{\mathrm{OB}} = 9 \end{cases}$

(2)　$\begin{cases} \mathrm{OA} = 2\sqrt{2} \\ \mathrm{OB} = \sqrt{6} \\ \overrightarrow{\mathrm{OA}} \cdot \overrightarrow{\mathrm{OB}} = -6 \end{cases}$

(3)　xy 座標平面上 $\begin{cases} \mathrm{A}(2, 4) \\ \mathrm{B}(3, 1) \end{cases}$

(4)　xy 座標平面上 $\begin{cases} \mathrm{A}(10, -6) \\ \mathrm{B}(-3, -5) \end{cases}$

2.　$\angle$AOB は何度か求めよ.（ S 級 1 分, A 級 2 分, B 級 3 分, C 級 5 分 ）

(1)　$\begin{cases} \mathrm{OA} = 8 \\ \mathrm{OB} = 4 \\ \overrightarrow{\mathrm{OA}} \cdot \overrightarrow{\mathrm{OB}} = -16 \end{cases}$

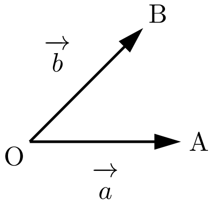
(2)　$\begin{cases} \mathrm{OA} = 2\sqrt{6} \\ \mathrm{OB} = 3\sqrt{3} \\ \overrightarrow{\mathrm{OA}} \cdot \overrightarrow{\mathrm{OB}} = 9\sqrt{6} \end{cases}$

(3)　xy 座標平面上 $\begin{cases} \mathrm{A}(-6, 2) \\ \mathrm{B}(2, 1) \end{cases}$

(4)　xy 座標平面上 $\begin{cases} \mathrm{A}(8, -6) \\ \mathrm{B}(-4, 3) \end{cases}$

反射テスト　ベクトル　内積から角度を求める　01　解答解説

1.　∠AOB は何度か求めよ.（ S 級 1 分, A 級 2 分, B 級 3 分, C 級 5 分 ）



★ベクトルの内積と角度

$$\begin{cases} \overrightarrow{\mathrm{OA}}\cdot\overrightarrow{\mathrm{OB}} = \mathrm{OA}\ \mathrm{OB}\ \cos\angle\mathrm{AOB} \iff \boldsymbol{\cos\angle\mathrm{AOB} = \dfrac{\overrightarrow{\mathrm{OA}}\cdot\overrightarrow{\mathrm{OB}}}{\mathrm{OA}\ \mathrm{OB}}} \\ \vec{a}\cdot\vec{b} = \left|\vec{a}\right|\left|\vec{b}\right|\cos\theta \iff \boldsymbol{\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{\left|\vec{a}\right|\left|\vec{b}\right|}} \end{cases}$$

(1)

$$\begin{cases} \mathrm{OA} = 3 \\ \mathrm{OB} = 6 \\ \overrightarrow{\mathrm{OA}}\cdot\overrightarrow{\mathrm{OB}} = 9 \end{cases}$$

$$\cos\angle\mathrm{AOB} = \frac{\overrightarrow{\mathrm{OA}}\cdot\overrightarrow{\mathrm{OB}}}{\mathrm{OA}\ \mathrm{OB}} = \frac{9}{3\times 6} = \frac{1}{2}$$

$\therefore$　$\boldsymbol{\angle\mathrm{AOB} = 60^\circ}$　**…答え**

(2)

$$\begin{cases} \mathrm{OA} = 2\sqrt{2} \\ \mathrm{OB} = \sqrt{6} \\ \overrightarrow{\mathrm{OA}}\cdot\overrightarrow{\mathrm{OB}} = -6 \end{cases}$$

$$\cos\angle\mathrm{AOB} = \frac{\overrightarrow{\mathrm{OA}}\cdot\overrightarrow{\mathrm{OB}}}{\mathrm{OA}\ \mathrm{OB}} = \frac{-6}{2\sqrt{2}\times\sqrt{6}} = -\frac{\sqrt{3}}{2}$$

$\therefore$　$\boldsymbol{\angle\mathrm{AOB} = 150^\circ}$　**…答え**

(3)　xy 座標平面上 $\begin{cases} \mathrm{A}(2,4) \\ \mathrm{B}(3,1) \end{cases}$

$\overrightarrow{\mathrm{OA}} = (2,4)$,　$\overrightarrow{\mathrm{OB}} = (3,1)$

$$\therefore \begin{cases} \overrightarrow{\mathrm{OA}}\cdot\overrightarrow{\mathrm{OB}} = 2\times 3 + 4\times 1 = 10 \\ \mathrm{OA} = \sqrt{2^2+4^2} = 2\sqrt{5} \\ \mathrm{OB} = \sqrt{3^2+1^2} = \sqrt{10} \end{cases}$$

$$\cos\angle\mathrm{AOB} = \frac{\overrightarrow{\mathrm{OA}}\cdot\overrightarrow{\mathrm{OB}}}{\mathrm{OA}\ \mathrm{OB}} = \frac{10}{2\sqrt{5}\times\sqrt{10}} = \frac{1}{\sqrt{2}}$$

$\therefore$　$\boldsymbol{\angle\mathrm{AOB} = 45^\circ}$　**…答え**

(4)　xy 座標平面上 $\begin{cases} \mathrm{A}(10,-6) \\ \mathrm{B}(-3,-5) \end{cases}$

$\overrightarrow{\mathrm{OA}} = (10,-6)$,　$\overrightarrow{\mathrm{OB}} = (-3,-5)$

$\therefore$　$\overrightarrow{\mathrm{OA}}\cdot\overrightarrow{\mathrm{OB}} = 10\times(-3) + (-6)\times(-5) = 0$

$\Rightarrow$　$\boldsymbol{\angle\mathrm{AOB} = 90^\circ}$　**…答え**

★内積が 0 ⇔ 垂直

2. $\angle$AOB は何度か求めよ.（S 級 1 分, A 級 2 分, B 級 3 分, C 級 5 分）

(1) $\begin{cases} \mathrm{OA} = 8 \\ \mathrm{OB} = 4 \\ \overrightarrow{\mathrm{OA}} \cdot \overrightarrow{\mathrm{OB}} = -16 \end{cases}$

$$\cos \angle \mathrm{AOB} = \frac{\overrightarrow{\mathrm{OA}} \cdot \overrightarrow{\mathrm{OB}}}{\mathrm{OA}\ \mathrm{OB}} = \frac{-16}{8 \times 4} = -\frac{1}{2}$$

$\therefore$ $\angle \boldsymbol{AOB} = \mathbf{120^\circ}$ **…答え**

(2) $\begin{cases} \mathrm{OA} = 2\sqrt{6} \\ \mathrm{OB} = 3\sqrt{3} \\ \overrightarrow{\mathrm{OA}} \cdot \overrightarrow{\mathrm{OB}} = 9\sqrt{6} \end{cases}$

$$\cos \angle \mathrm{AOB} = \frac{\overrightarrow{\mathrm{OA}} \cdot \overrightarrow{\mathrm{OB}}}{\mathrm{OA}\ \mathrm{OB}} = \frac{9\sqrt{6}}{2\sqrt{6} \times 3\sqrt{3}} = \frac{\sqrt{3}}{2}$$

$\therefore$ $\angle \boldsymbol{AOB} = \mathbf{30^\circ}$ **…答え**

(3) xy 座標平面上 $\begin{cases} \mathrm{A}(-6, 2) \\ \mathrm{B}(2, 1) \end{cases}$

$\overrightarrow{\mathrm{OA}} = (-6, 2)$, $\overrightarrow{\mathrm{OB}} = (2, 1)$

$\therefore \begin{cases} \overrightarrow{\mathrm{OA}} \cdot \overrightarrow{\mathrm{OB}} = -6 \times 2 + 2 \times 1 = -10 \\ \mathrm{OA} = \sqrt{(-6)^2 + 2^2} = 2\sqrt{10} \\ \mathrm{OB} = \sqrt{2^2 + 1^2} = \sqrt{5} \end{cases}$

$$\cos \angle \mathrm{AOB} = \frac{\overrightarrow{\mathrm{OA}} \cdot \overrightarrow{\mathrm{OB}}}{\mathrm{OA}\ \mathrm{OB}} = \frac{-10}{2\sqrt{10} \times \sqrt{5}} = -\frac{1}{\sqrt{2}}$$

$\therefore$ $\angle \boldsymbol{AOB} = \mathbf{135^\circ}$ **…答え**

(4) xy 座標平面上 $\begin{cases} \mathrm{A}(8, -6) \\ \mathrm{B}(-4, 3) \end{cases}$

$\overrightarrow{\mathrm{OA}} = (8, -6)$, $\overrightarrow{\mathrm{OB}} = (-4, 3)$

$\therefore \begin{cases} \overrightarrow{\mathrm{OA}} \cdot \overrightarrow{\mathrm{OB}} = 8 \times (-4) + (-6) \times 3 = -50 \\ \mathrm{OA} = \sqrt{8^2 + (-6)^2} = 10 \\ \mathrm{OB} = \sqrt{(-4)^2 + 3^2} = 5 \end{cases}$

$$\cos \angle \mathrm{AOB} = \frac{\overrightarrow{\mathrm{OA}} \cdot \overrightarrow{\mathrm{OB}}}{\mathrm{OA}\ \mathrm{OB}} = \frac{-50}{10 \times 5} = -1$$

$\therefore$ $\angle \boldsymbol{AOB} = \mathbf{180^\circ}$ **…答え**

★座標平面のイメージができれば計算なしで求められる.