反射テスト　数列　部分分数分解　02

1.　次の計算をせよ. (S 級 2 分 30 秒, A 級 4 分, B 級 6 分, C 級 9 分)

(1)　$\displaystyle\sum_{k=1}^{n} \frac{1}{k(k+1)}$

(2)　$\displaystyle\sum_{k=1}^{n} \frac{1}{k(k+2)}$

2. 次の計算をせよ. (S 級 2 分 30 秒, A 級 4 分, B 級 6 分, C 級 9 分)

(1) $\displaystyle\sum_{k=1}^{n} \frac{1}{(2k-1)(2k+1)}$

(2) $\displaystyle\sum_{k=1}^{n} \frac{1}{(k+1)(k+3)}$

反射テスト　数列　部分分数分解　02　解答解説

1.　次の計算をせよ.（S 級 2 分 30 秒, A 級 4 分, B 級 6 分, C 級 9 分）

★ 部分分数分解　次の式変形を **部分分数分解** という. A と B はいざとなれば恒等式として導く.

$$\frac{1}{(a_1x+a_2)(b_1x+b_2)}=\frac{A}{a_1x+a_2}+\frac{B}{b_1x+b_2}$$

(1)　$\displaystyle\sum_{k=1}^{n}\frac{1}{k(k+1)}$

$$=\sum_{k=1}^{n}\left(\frac{1}{k}-\frac{1}{k+1}\right)\quad\leftarrow\bigstar$$
$$=\left(\frac{1}{1}-\cancel{\frac{1}{2}}\right)+\left(\cancel{\frac{1}{2}}-\cancel{\frac{1}{3}}\right)+\left(\cancel{\frac{1}{3}}-\cancel{\frac{1}{4}}\right)+\cdots+\left(\cancel{\frac{1}{n-1}}-\cancel{\frac{1}{n}}\right)+\left(\cancel{\frac{1}{n}}-\frac{1}{n+1}\right)\quad\leftarrow\text{相殺}$$
$$=\boldsymbol{1-\frac{1}{n+1}}\quad\boldsymbol{(n=1,2,3,\cdots)}$$
$$=\boldsymbol{\frac{n}{n+1}}\quad\boldsymbol{(n=1,2,3,\cdots)}$$

★部分分数分解
$$\frac{1}{k(k+1)}=\frac{(k+1)-k}{k(k+1)}=\frac{k+1}{k(k+1)}-\frac{k}{k(k+1)}=\frac{1}{k}-\frac{1}{k+1}$$

(2)　$\displaystyle\sum_{k=1}^{n}\frac{1}{k(k+2)}$

$$=\frac{1}{2}\sum_{k=1}^{n}\left(\frac{1}{k}-\frac{1}{k+2}\right)\quad\leftarrow\bigstar$$
$$=\frac{1}{2}\left\{\left(\frac{1}{1}-\cancel{\frac{1}{3}}\right)+\left(\frac{1}{2}-\cancel{\frac{1}{4}}\right)+\left(\cancel{\frac{1}{3}}-\cancel{\frac{1}{5}}\right)+\cdots+\left(\cancel{\frac{1}{n-1}}-\frac{1}{n+1}\right)+\left(\cancel{\frac{1}{n}}-\frac{1}{n+2}\right)\right\}\quad\leftarrow\text{☆ 1 個とばしの相殺}$$
$$=\frac{1}{2}\left(1+\frac{1}{2}-\frac{1}{n+1}-\frac{1}{n+2}\right)$$
$$=\boldsymbol{\frac{3}{4}-\frac{1}{2(n+1)}-\frac{1}{2(n+2)}}$$
$$=\frac{3(n+1)(n+2)-2(n+2)-2(n+1)}{4(n+1)(n+2)}$$
$$=\boldsymbol{\frac{3n^2+5n}{4(n+1)(n+2)}}\quad\boldsymbol{(n=1,2,3,\cdots)}$$
$$=\boldsymbol{\frac{n(3n+5)}{4(n+1)(n+2)}}\quad\boldsymbol{(n=1,2,3,\cdots)}$$

★部分分数分解
$$\frac{1}{k(k+2)}=\frac{1}{2}\cdot\frac{(k+2)-k}{k(k+2)}=\frac{1}{2}\left\{\frac{k+2}{k(k+2)}-\frac{k}{k(k+2)}\right\}=\frac{1}{2}\left(\frac{1}{k}-\frac{1}{k+2}\right)$$

☆答えは最後の因数分解の形がよい.

2. 次の計算をせよ.（S 級 2 分 30 秒, A 級 4 分, B 級 6 分, C 級 9 分）

(1) $\displaystyle\sum_{k=1}^{n}\frac{1}{(2k-1)(2k+1)}$

$$
\begin{aligned}
&= \frac{1}{2}\sum_{k=1}^{n}\frac{(2k+1)-(2k-1)}{(2k-1)(2k+1)} \quad \leftarrow\bigstar\\
&= \frac{1}{2}\sum_{k=1}^{n}\left(\frac{1}{2k-1}-\frac{1}{2k+1}\right) \quad \leftarrow\bigstar\\
&= \frac{1}{2}\left\{\left(\frac{1}{1}-\cancel{\frac{1}{3}}\right)+\left(\cancel{\frac{1}{3}}-\cancel{\frac{1}{5}}\right)+\left(\cancel{\frac{1}{5}}-\cancel{\frac{1}{7}}\right)+\cdots+\left(\cancel{\frac{1}{2n-3}}-\cancel{\frac{1}{2n-1}}\right)+\left(\cancel{\frac{1}{2n-1}}-\frac{1}{2n+1}\right)\right\} \quad \leftarrow\text{相殺}\\
&= \boldsymbol{\frac{1}{2}-\frac{1}{2(2n+1)}} \quad \boldsymbol{(n=1,2,3,\cdots)}\\
&= \boldsymbol{\frac{n}{2n+1}} \quad \boldsymbol{(n=1,2,3,\cdots)}
\end{aligned}
$$

★部分分数分解

$$
\begin{aligned}
\frac{1}{(2k-1)(2k+1)} &= \frac{1}{2}\left\{\frac{(2k+1)-(2k-1)}{(2k-1)(2k+1)}\right\}\\
&= \frac{1}{2}\left\{\frac{2k+1}{(2k-1)(2k+1)}-\frac{2k-1}{(2k-1)(2k+1)}\right\} = \frac{1}{2}\left(\frac{1}{2k-1}-\frac{1}{2k+1}\right)
\end{aligned}
$$

(2) $\displaystyle\sum_{k=1}^{n}\frac{1}{(k+1)(k+3)}$

$$
\begin{aligned}
&= \frac{1}{2}\sum_{k=1}^{n}\left(\frac{1}{k+1}-\frac{1}{k+3}\right) \quad \leftarrow\bigstar\\
&= \frac{1}{2}\left\{\left(\frac{1}{2}-\cancel{\frac{1}{4}}\right)+\left(\frac{1}{3}-\cancel{\frac{1}{5}}\right)+\left(\cancel{\frac{1}{4}}-\cancel{\frac{1}{6}}\right)+\cdots+\left(\cancel{\frac{1}{n}}-\frac{1}{n+2}\right)+\left(\cancel{\frac{1}{n+1}}-\frac{1}{n+3}\right)\right\} \quad \leftarrow\text{☆ 1 個とばしの相殺}\\
&= \frac{1}{2}\left(\frac{1}{2}+\frac{1}{3}-\frac{1}{n+2}-\frac{1}{n+3}\right)\\
&= \boldsymbol{\frac{5}{12}-\frac{1}{2(n+2)}-\frac{1}{2(n+3)}}\\
&= \frac{5(n+2)(n+3)-6(n+3)-6(n+2)}{12(n+2)(n+3)}\\
&= \boldsymbol{\frac{5n^2+13n}{12(n+2)(n+3)}} \quad \boldsymbol{(n=1,2,3,\cdots)}\\
&= \boldsymbol{\frac{n(5n+13)}{12(n+2)(n+3)}} \quad \boldsymbol{(n=1,2,3,\cdots)}
\end{aligned}
$$

★部分分数分解

$$
\frac{1}{(k+1)(k+3)} = \frac{1}{2}\cdot\frac{(k+3)-(k+1)}{(k+1)(k+3)} = \frac{1}{2}\left\{\frac{k+3}{(k+1)(k+3)}-\frac{k+1}{(k+1)(k+3)}\right\} = \frac{1}{2}\left(\frac{1}{k+1}-\frac{1}{k+3}\right)
$$

☆答えは最後の因数分解の形がよい.