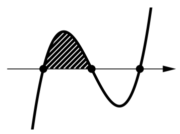
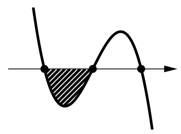
反射テスト　積分　定積分　3 次関数と面積の公式　01

1.　斜線部分の面積を求めよ. (S 級 3 分, A 級 5 分, B 級 7 分, C 級 10 分)

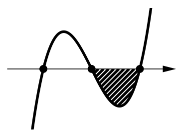
(1)　$y = x^3 - x$



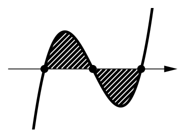
(2)　$y = 6x - 6x^3$



(3)　$y = 8x^3 - 2x$

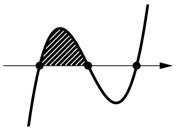


(4)　$y = x^3 - 2x^2 - x + 2$

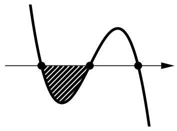


2. 斜線部分の面積を求めよ.（ S 級 4 分, A 級 6 分, B 級 9 分, C 級 12 分 ）

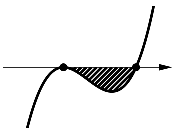
(1) $y = 4x^3 - 4x^2 - 8x$



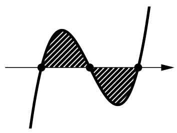
(2) $y = 8x - 2x^3$



(3) $y = 6x^3 - 2x^2$



(4) $y = 2x^3 + 3x^2 - 11x - 6$



反射テスト　積分　定積分　3次関数と面積　01　解答解説

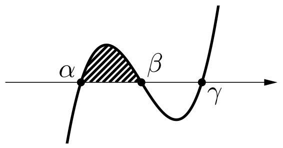
★ 公式 0　$\displaystyle\int_{-t}^{t}$ **奇関数** $\boldsymbol{dx = 0}$　$\displaystyle\int_{-t}^{t}$ **偶関数** $\boldsymbol{dx = 2}\displaystyle\int_{0}^{t}$ **偶関数** $\boldsymbol{dx}$

★ 公式 1　$\displaystyle\boldsymbol{\int_{\alpha}^{\beta}(x-\alpha)(x-\beta)(x-\gamma)\,dx = \frac{1}{12}(\beta-\alpha)^3(2\gamma-\alpha-\beta)}$　←下の (ア), (イ) どちらに対しても成立

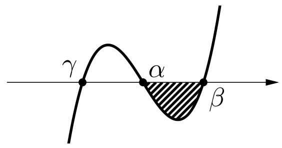
★ 公式 2　$\displaystyle\boldsymbol{\int_{\alpha}^{\beta}a(x-\alpha)(x-\beta)(x-\gamma)\,dx = \frac{a}{12}(\beta-\alpha)^3(2\gamma-\alpha-\beta)}$　←公式 1 の定数 a 倍

公式 1 と 2 は絶対に覚えなければならない公式ではないが，計算がとても楽になる．

(ア)　$y=(x-\alpha)(x-\beta)(x-\gamma)$　ただし $\alpha \leqq \beta \leqq \gamma$

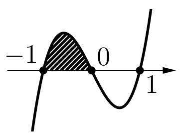


(イ)　$y=(x-\gamma)(x-\alpha)(x-\beta)$　ただし $\gamma \leqq \alpha \leqq \beta$



1.　斜線部分の面積を求めよ．(S 級 3 分，A 級 5 分，B 級 7 分，C 級 10 分)

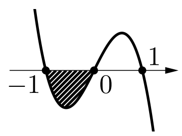
(1)　$y=x^3-x$



$y=(x+1)x(x-1)$
$\therefore$ x 切片は -1 , 0 , 1

$$\int_{-1}^{0}\{(x+1)x(x-1)-0\}\,dx$$
$$=\frac{1}{12}\cdot\{0-(-1)\}^3\cdot\{2\cdot 1-(-1)-0\}$$
$$=\boldsymbol{\frac{1}{4}}$$ **…答え**

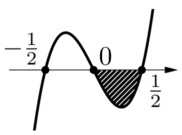
(2)　$y=6x-6x^3$



$y=-6x(x^2-1)$
$\Leftrightarrow$　$y=-6(x+1)x(x-1)$
$\therefore$ x 切片は -1 , 0 , 1

$$\int_{-1}^{0}\{0-(6x-6x^3)\}\,dx$$
$$=+6\int_{-1}^{0}(x+1)x(x-1)\,dx$$
$$=6\cdot\frac{1}{12}\cdot\{0-(-1)\}^3\cdot\{2\cdot 1-(-1)-0\}$$
$$=\boldsymbol{\frac{3}{2}}$$ **…答え**

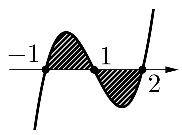
(3)　$y=8x^3-2x$



$y=2x(4x^2-1)$
$\Leftrightarrow$　$y=2x(2x+1)(2x-1)$
$\Leftrightarrow$　$y=8\cdot\left(x+\frac{1}{2}\right)x\left(x-\frac{1}{2}\right)$
$\therefore$ x 切片は $-\frac{1}{2}$, 0 , $\frac{1}{2}$

$$\int_{0}^{\frac{1}{2}}0\,dx-\int_{0}^{\frac{1}{2}}(8x^3-2x)\,dx$$
$$=-8\int_{0}^{\frac{1}{2}}\left(x+\tfrac{1}{2}\right)x\left(x-\tfrac{1}{2}\right)dx$$
$$=-8\cdot\frac{1}{12}\cdot\left(\tfrac{1}{2}-0\right)^3\cdot\left\{2\cdot\left(-\tfrac{1}{2}\right)-0-\tfrac{1}{2}\right\}$$
$$=\boldsymbol{\frac{1}{8}}$$ **…答え**

(4)　$y=x^3-2x^2-x+2$

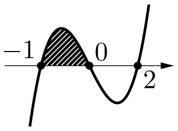


$y=x^2(x-2)-(x-2)$
$\Leftrightarrow$　$y=(x+1)(x-1)(x-2)$
$\therefore$ x 切片は -1 , 1 , 2

$$\int_{-1}^{1}(x+1)(x-1)(x-2)\,dx$$
$$-\int_{1}^{2}(x+1)(x-1)(x-2)\,dx$$
$$=\tfrac{1}{12}\cdot 2^3\cdot\{2\cdot 2-(-1)-1\}-\tfrac{1}{12}\cdot 1^3\cdot\{2\cdot(-1)-1-2\}$$
$$=\frac{32+5}{12}=\boldsymbol{\frac{37}{12}}$$ **…答え**

2. 斜線部分の面積を求めよ.(S 級 4 分, A 級 6 分, B 級 9 分, C 級 12 分)

(1) $y = 4x^3 - 4x^2 - 8x$



$\Leftrightarrow$ $y = 4x(x^2 - x - 2)$

$y = 4(x+1)x(x-2)$

$\therefore$ x 切片は -1 , 0 , 2

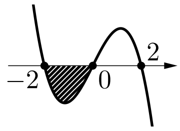
$$\int_{-1}^{0} (4x^3 - 4x^2 - 8x)\,dx$$

$$= 4\int_{-1}^{0} (x+1)x(x-2)\,dx$$

$$= 4 \cdot \frac{1}{12} \cdot \{0 - (-1)\}^3 \cdot \{2 \cdot 2 - (-1) - 0\}$$

$$= \mathbf{\frac{5}{3}}$$ **…答え**

(2) $y = 8x - 2x^3$



$\Leftrightarrow$ $y = -2x(x^2 - 4)$

$y = -2(x+2)x(x-2)$

$\therefore$ x 切片は -2 , 0 , 2

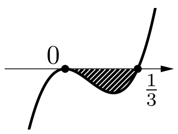
$$\int_{-2}^{0} \{0 - (8x - 2x^3)\}\,dx$$

$$= +2\int_{-2}^{0} (x+2)x(x-2)\,dx$$

$$= 2 \cdot \frac{1}{12} \cdot \{0 - (-2)\}^3 \cdot \{2 \cdot 2 - (-2) - 0\}$$

$$= \mathbf{8}$$ **…答え**

(3) $y = 6x^3 - 2x^2$



$\Leftrightarrow$ $y = 2x^2(3x - 1)$

$y = 6x^2\left(x - \frac{1}{3}\right)$

$\therefore$ x 切片は 0 , $\frac{1}{3}$

$$\int_{0}^{\frac{1}{3}} 0\,dx - \int_{0}^{\frac{1}{3}} (6x^3 - 2x^2)\,dx$$

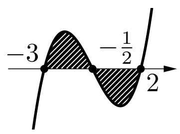
$$= -6\int_{0}^{\frac{1}{3}} x^2\left(x - \frac{1}{3}\right)dx$$

$$= -6 \cdot \frac{1}{12} \cdot \left(\frac{1}{3} - 0\right)^3 \cdot \left\{2 \cdot 0 - 0 - \frac{1}{3}\right\}$$

$$= -6 \cdot \frac{1}{12} \cdot \frac{1}{27} \cdot \left(-\frac{1}{3}\right)$$

$$= \mathbf{\frac{1}{162}}$$ **…答え**

(4) $y = 2x^3 + 3x^2 - 11x - 6$



$\Leftrightarrow$ $y = (x-2)(2x^2 + 7x + 3)$

$y = (x+3)(2x+1)(x-2)$

$\therefore$ x 切片は -3 , $-\frac{1}{2}$, 2

$$\int_{-3}^{-\frac{1}{2}} 2(x+3)\left(x + \frac{1}{2}\right)(x-2)\,dx - \int_{-\frac{1}{2}}^{2} 2(x+3)\left(x + \frac{1}{2}\right)(x-2)\,dx$$

$$= \frac{2}{12} \cdot \left(\frac{5}{2}\right)^3 \cdot \left\{2 \cdot 2 - (-3) - \left(-\frac{1}{2}\right)\right\} - \frac{2}{12} \cdot \left(\frac{5}{2}\right)^3 \cdot \left\{2 \cdot (-3) - \left(-\frac{1}{2}\right) - 2\right\}$$

$$= \frac{625}{32} + \frac{625}{32}$$

$$= \mathbf{\frac{625}{16}}$$ **…答え**