

反射テスト 2次方程式 解と係数の関係 01

1. 次の2次方程式の解を α, β とする. $\langle \quad \rangle$ の中の式の値を求めよ. (S級1分, A級1分20秒, B級2分, C級3分)

(1) $x^2 + 5x - 3 = 0$ $\langle \alpha + \beta \rangle$

(2) $3x^2 - 14x + 2 = 0$ $\langle \alpha\beta \rangle$

(3) $x^2 - 11x - 23 = 0$ $\langle \alpha + \beta - \alpha\beta \rangle$

(4) $x^2 - 4x + 2 = 0$ $\langle \alpha^2 + \beta^2 \rangle$

(5) $5x^2 + 6x - 10 = 0$ $\langle \alpha^2 + 2\alpha\beta + \beta^2 \rangle$

(6) $3x^2 - 9x + 5 = 0$ $\langle \alpha^2 - \alpha\beta + \beta^2 \rangle$

2. 次の2次方程式の解を α, β とする. $\langle \quad \rangle$ の中の式の値を求めよ.

(S 級 1 分 10 秒, A 級 1 分 30 秒, B 級 2 分 20 秒, C 級 3 分 30 秒)

(1) $x^2 - 4x - 6 = 0$ $\langle \alpha\beta \rangle$

(2) $3x^2 + 4x - 5 = 0$ $\langle \alpha + \beta \rangle$

(3) $x^2 - 17x - 23 = 0$ $\langle \alpha - 2\alpha\beta + \beta \rangle$

(4) $x^2 - 7x + 3 = 0$ $\langle \alpha^2 + \beta^2 \rangle$

(5) $2x^2 + 5x - 8 = 0$ $\langle \alpha^2 + \alpha\beta + \beta^2 \rangle$

(6) $2x^2 - 9x + 6 = 0$ $\langle (\alpha - \beta)^2 \rangle$

反射テスト 2次方程式 解と係数の関係 01 解答解説

1. 次の2次方程式の解を α, β とする. $< >$ 中の式の値を求めよ. (S級1分, A級1分20秒, B級2分, C級3分)

★ 解と係数の関係

2次方程式 $ax^2 + bx + c = 0$ の解を α, β としたとき,

$$\alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a}$$

(1) $x^2 + 5x - 3 = 0$ $< \alpha + \beta >$

$$\alpha + \beta = -\frac{5}{1} = -5$$

(2) $3x^2 - 14x + 2 = 0$ $< \alpha\beta >$

$$\alpha\beta = \frac{2}{3}$$

(3) $x^2 - 11x - 23 = 0$ $< \alpha + \beta - \alpha\beta >$

$$\alpha + \beta = -\frac{-11}{1} = 11$$

$$\alpha\beta = \frac{-23}{1} = -23$$

$$\begin{aligned} \alpha + \beta - \alpha\beta &= 11 - (-23) \\ &= 11 + 23 \\ &= \mathbf{34} \end{aligned}$$

(4) $x^2 - 4x + 2 = 0$ $< \alpha^2 + \beta^2 >$

$$\alpha + \beta = -\frac{-4}{1} = 4$$

$$\alpha\beta = \frac{2}{1} = 2$$

$$\begin{aligned} \alpha^2 + \beta^2 &= (\alpha + \beta)^2 - 2\alpha\beta \\ &= 4^2 - 2 \times 2 \\ &= \mathbf{12} \end{aligned}$$

(5) $5x^2 + 6x - 10 = 0$ $< \alpha^2 + 2\alpha\beta + \beta^2 >$

$$\alpha + \beta = -\frac{6}{5}$$

$$\alpha\beta = \frac{-10}{5} = -2$$

$$\begin{aligned} \alpha^2 + 2\alpha\beta + \beta^2 &= (\alpha + \beta)^2 \\ &= \left(-\frac{6}{5}\right)^2 \\ &= \frac{\mathbf{36}}{\mathbf{25}} \end{aligned}$$

(6) $3x^2 - 9x + 5 = 0$ $< \alpha^2 - \alpha\beta + \beta^2 >$

$$\alpha + \beta = -\frac{-9}{3} = 3$$

$$\alpha\beta = \frac{5}{3}$$

$$\begin{aligned} \alpha^2 - \alpha\beta + \beta^2 &= (\alpha + \beta)^2 - 3\alpha\beta \\ &= 3^2 - 3 \times \frac{5}{3} \\ &= 9 - 5 = \mathbf{4} \end{aligned}$$

2. 次の2次方程式の解を α, β とする. $\langle \quad \rangle$ の中の式の値を求めよ.

(S級1分10秒, A級1分30秒, B級2分20秒, C級3分30秒)

(1) $x^2 - 4x - 6 = 0$ $\langle \alpha\beta \rangle$

$$\alpha\beta = \frac{-6}{1} = -6$$

(2) $3x^2 + 4x - 5 = 0$ $\langle \alpha + \beta \rangle$

$$\alpha + \beta = -\frac{4}{3}$$

(3) $x^2 - 17x - 23 = 0$ $\langle \alpha - 2\alpha\beta + \beta \rangle$

$$\alpha + \beta = -\frac{-17}{1} = 17$$

$$\alpha\beta = \frac{-23}{1} = -23$$

$$\begin{aligned} \alpha + \beta - 2\alpha\beta \\ &= 17 - 2 \times (-23) \\ &= 17 + 46 \\ &= \mathbf{63} \end{aligned}$$

(4) $x^2 - 7x + 3 = 0$ $\langle \alpha^2 + \beta^2 \rangle$

$$\alpha + \beta = -\frac{-7}{1} = 7$$

$$\alpha\beta = \frac{3}{1} = 3$$

$$\begin{aligned} \alpha^2 + \beta^2 \\ &= (\alpha + \beta)^2 - 2\alpha\beta \\ &= 7^2 - 2 \times 3 \\ &= 49 - 6 \\ &= \mathbf{43} \end{aligned}$$

(5) $2x^2 + 5x - 8 = 0$ $\langle \alpha^2 + \alpha\beta + \beta^2 \rangle$

$$\alpha + \beta = -\frac{5}{2}$$

$$\alpha\beta = \frac{-8}{2} = -4$$

$$\begin{aligned} \alpha^2 + \alpha\beta + \beta^2 \\ &= (\alpha + \beta)^2 - \alpha\beta \\ &= \left(-\frac{5}{2}\right)^2 - (-4) \\ &= \frac{25}{4} + 4 \\ &= \frac{\mathbf{41}}{4} \end{aligned}$$

(6) $2x^2 - 9x + 6 = 0$ $\langle (\alpha - \beta)^2 \rangle$

$$\alpha + \beta = -\frac{-9}{2} = \frac{9}{2}$$

$$\alpha\beta = \frac{6}{2} = 3$$

$$\begin{aligned} (\alpha - \beta)^2 \\ &= \alpha^2 - 2\alpha\beta + \beta^2 \\ &= (\alpha + \beta)^2 - 4\alpha\beta \\ &= \left(\frac{9}{2}\right)^2 - 4 \times 3 \\ &= \frac{81}{4} - 12 \\ &= \frac{\mathbf{33}}{4} \end{aligned}$$