

反射テスト 1次方程式 連立方程式 難問いろいろ 02

1. 次の連立方程式を解け。(S級1分20秒, A級2分30秒, B級4分, C級6分)

$$(1) \begin{cases} \frac{1}{x+y} + \frac{1}{x-y} = 9 \\ \frac{1}{x+y} - \frac{1}{x-y} = 5 \end{cases}$$

$$(2) \begin{cases} 123x + 789y = 456 \\ 321x + 987y = 654 \end{cases}$$

2. 次の連立方程式を解け。(S級2分, A級3分30秒, B級5分30秒, C級8分)

$$(1) \begin{cases} \frac{1}{x+y} + \frac{1}{y+z} = 3 \\ \frac{1}{y+z} + \frac{1}{z+x} = 4 \\ \frac{1}{z+x} + \frac{1}{x+y} = 5 \end{cases} \quad (2) \begin{cases} 456x + 789y = 123 \\ 654x + 987y = 321 \end{cases}$$

反射テスト 1次方程式 連立方程式 難問いろいろ 02 解答解説

1. 次の連立方程式を解け。(S級1分20秒, A級2分30秒, B級4分, C級6分)

$$(1) \begin{cases} \frac{1}{x+y} + \frac{1}{x-y} = 9 & \dots \textcircled{1} \\ \frac{1}{x+y} - \frac{1}{x-y} = 5 & \dots \textcircled{2} \end{cases}$$

☆ $\frac{1}{x+y} = a, \frac{1}{x-y} = b$ とおく.

$$\begin{cases} a+b=9 & \dots \textcircled{3} \\ a-b=5 & \dots \textcircled{4} \end{cases}$$

$$\begin{array}{r} \textcircled{3} \quad a + b = 9 \\ +) \textcircled{4} \quad a - b = 5 \\ \hline 2a = 14 \\ a = 7 \end{array}$$

③ に代入して $7+b=9 \Leftrightarrow b=2$

$(a, b) = (7, 2)$

$$\Rightarrow \begin{cases} x+y = \frac{1}{7} & \dots \textcircled{5} \\ x-y = \frac{1}{2} & \dots \textcircled{6} \end{cases}$$

$$\begin{array}{r} \textcircled{5} \quad x + y = \frac{1}{7} \\ +) \textcircled{6} \quad x - y = \frac{1}{2} \\ \hline 2x = \frac{9}{14} \\ x = \frac{9}{28} \end{array}$$

⑤ に代入して,

$$\frac{9}{28} + y = \frac{1}{7}$$

$$9 + 28y = 4 \quad \leftarrow \text{両辺} \times 28$$

$$y = -\frac{5}{28}$$

$$(x, y) = \left(\frac{9}{28}, -\frac{5}{28} \right) \quad \dots \text{答え}$$

$$(2) \begin{cases} 123x + 789y = 456 & \dots \textcircled{1} \\ 321x + 987y = 654 & \dots \textcircled{2} \end{cases}$$

★係数が大きいが対称的な数なので工夫できる.

$$\begin{array}{r} \textcircled{1} \quad 123x + 789y = 456 \\ +) \textcircled{2} \quad 321x + 987y = 654 \\ \hline 444x + 1776y = 1110 \\ 2x + 8y = 5 \quad \dots \textcircled{3} \end{array}$$

$$\begin{array}{r} \textcircled{1} \quad 123x + 789y = 456 \\ -) \textcircled{2} \quad 321x + 987y = 654 \\ \hline -198x - 198y = -198 \\ x + y = 1 \quad \dots \textcircled{4} \end{array}$$

$$\begin{array}{r} \textcircled{3} \quad 2x + 8y = 5 \\ -) \textcircled{4} \times 2 \quad 2x + 2y = 2 \\ \hline -6y = 3 \\ + \quad y = \frac{1}{2} \end{array}$$

④ に代入して,

$$\begin{array}{r} x + \frac{1}{2} = 1 \\ x = \frac{1}{2} \end{array}$$

$$(x, y) = \left(\frac{1}{2}, \frac{1}{2} \right) \quad \dots \text{答え}$$

2. 次の連立方程式を解け。(S級2分, A級3分30秒, B級5分30秒, C級8分)

$$(1) \begin{cases} \frac{1}{x+y} + \frac{1}{y+z} = 3 & \dots\textcircled{1} \\ \frac{1}{y+z} + \frac{1}{z+x} = 4 & \dots\textcircled{2} \\ \frac{1}{z+x} + \frac{1}{x+y} = 5 & \dots\textcircled{3} \end{cases}$$

☆ $\frac{1}{x+y} = a$, $\frac{1}{y+z} = b$, $\frac{1}{z+x} = c$ とおく.

$$\begin{cases} a+b=3 & \dots\textcircled{4} \\ b+c=4 & \dots\textcircled{5} \\ c+a=5 & \dots\textcircled{6} \end{cases}$$

$$\begin{array}{r} \textcircled{3} \quad a + b = 3 \\ \textcircled{4} \quad \quad b + c = 4 \\ +) \textcircled{5} \quad a \quad + c = 5 \\ \hline 2a + 2b + 2c = 12 \\ a + b + c = 6 \quad \dots\textcircled{7} \end{array}$$

$$\begin{cases} \textcircled{7} - \textcircled{4} & \Rightarrow c = 3 \\ \textcircled{7} - \textcircled{5} & \Rightarrow a = 2 \\ \textcircled{7} - \textcircled{6} & \Rightarrow b = 1 \end{cases}$$

$$\Rightarrow \begin{cases} \frac{1}{x+y} = 2 & \Rightarrow x+y = \frac{1}{2} \quad \dots\textcircled{8} \\ \frac{1}{y+z} = 1 & \Rightarrow y+z = 1 \quad \dots\textcircled{9} \\ \frac{1}{z+x} = 3 & \Rightarrow z+x = \frac{1}{3} \quad \dots\textcircled{10} \end{cases}$$

$$\begin{array}{r} \textcircled{3} \quad x + y = \frac{1}{2} \\ \textcircled{4} \quad \quad y + z = 1 \\ +) \textcircled{5} \quad x \quad + z = \frac{1}{3} \\ \hline 2x + 2y + 2z = \frac{11}{6} \\ x + y + z = \frac{11}{12} \quad \dots\textcircled{11} \end{array}$$

$$\begin{cases} \textcircled{11} - \textcircled{8} & \Rightarrow z = \frac{5}{12} \\ \textcircled{11} - \textcircled{9} & \Rightarrow x = -\frac{1}{12} \\ \textcircled{11} - \textcircled{10} & \Rightarrow y = \frac{7}{12} \end{cases}$$

$$(x, y, z) = \left(-\frac{1}{12}, \frac{7}{12}, \frac{5}{12}\right) \quad \dots\text{答え}$$

$$(2) \begin{cases} 456x + 789y = 123 & \dots\textcircled{1} \\ 654x + 987y = 321 & \dots\textcircled{2} \end{cases}$$

★係数が大きいが対称的な数なので工夫できる.

$$\begin{array}{r} \textcircled{1} \quad 456x + 789y = 123 \\ +) \textcircled{2} \quad 654x + 987y = 321 \\ \hline 1110x + 1776y = 444 \\ 5x + 8y = 2 \quad \dots\textcircled{3} \\ \textcircled{1} \quad 456x + 789y = 123 \\ -) \textcircled{2} \quad 654x + 987y = 321 \\ \hline -198x - 198y = -198 \\ x + y = 1 \quad \dots\textcircled{4} \\ \textcircled{3} \quad 5x + 8y = 2 \\ -) \textcircled{4} \times 5 \quad 5x + 5y = 5 \\ \hline -3y = -3 \\ + y = -1 \end{array}$$

④に代入して,

$$\begin{aligned} x + (-1) &= 1 \\ x &= 2 \end{aligned}$$

$$(x, y) = (2, -1) \quad \dots\text{答え}$$